Eigenmode Expansion Method to Analyze Wave Propagation in Twisted Optical Waveguides

IPS Conference 2021

Fyodor Morozko
Ben Gurion University, Belarusian State University

Introduction

- Recent developments in integrated silicon photonic devices have demonstrated the remarkable capabilities of the **silicon-on-insulator** (SOI) platform to implement a wide range of applications, such as *optical interconnects*, *nonlinear photonics*, *biological sensors*, and *microspectrometers*.

- However, silicon waveguide has **large structural birefringence** that causes polarization-mode dispersion, polarization-dependent loss, and polarization-dependent wavelength characteristics.

- The **polarization-dependent characteristics limit the application** of silicon photonics devices.

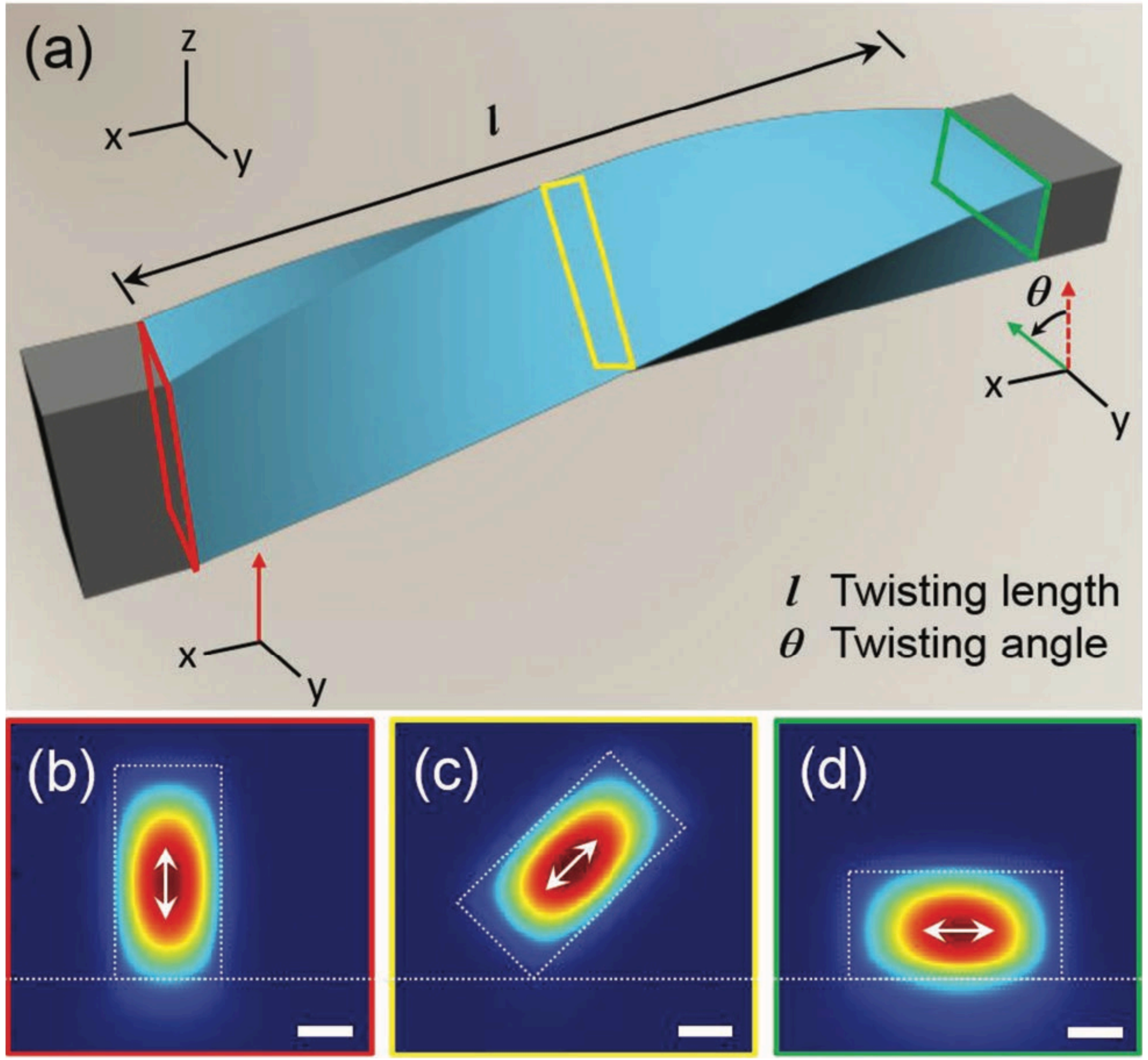
- As a result, **most of the photonic devices in SOI have been designed for a single mode and a single polarization**, typically TE.

- To *mimic* polarization independent operation, **polarization diversity schemes** are typically used: the light with arbitrary polarization from the source is passed through **polarization splitters** and **polarization rotators**.

[1] J. Zhang, *et al.*, Silicon-Waveguide-Based Mode Evolution Polarization Rotator, IEEE J. Select. Topics Quantum Electron. **16**, 53 (2010)

Polarization Rotator Based on Twisted Waveguides

- Recently, integrated polarization rotator based on a twisted waveguide was proposed [1].
- Polarization rotation in this scheme is based on adiabatic mode conversion.
- Advantages:
  - ✓ Low insertion losses
  - ✓ High conversion efficiencies
  - ✓ Broad bandwidth operation
  - ✓ Relaxed requirement on fabrication accuracy

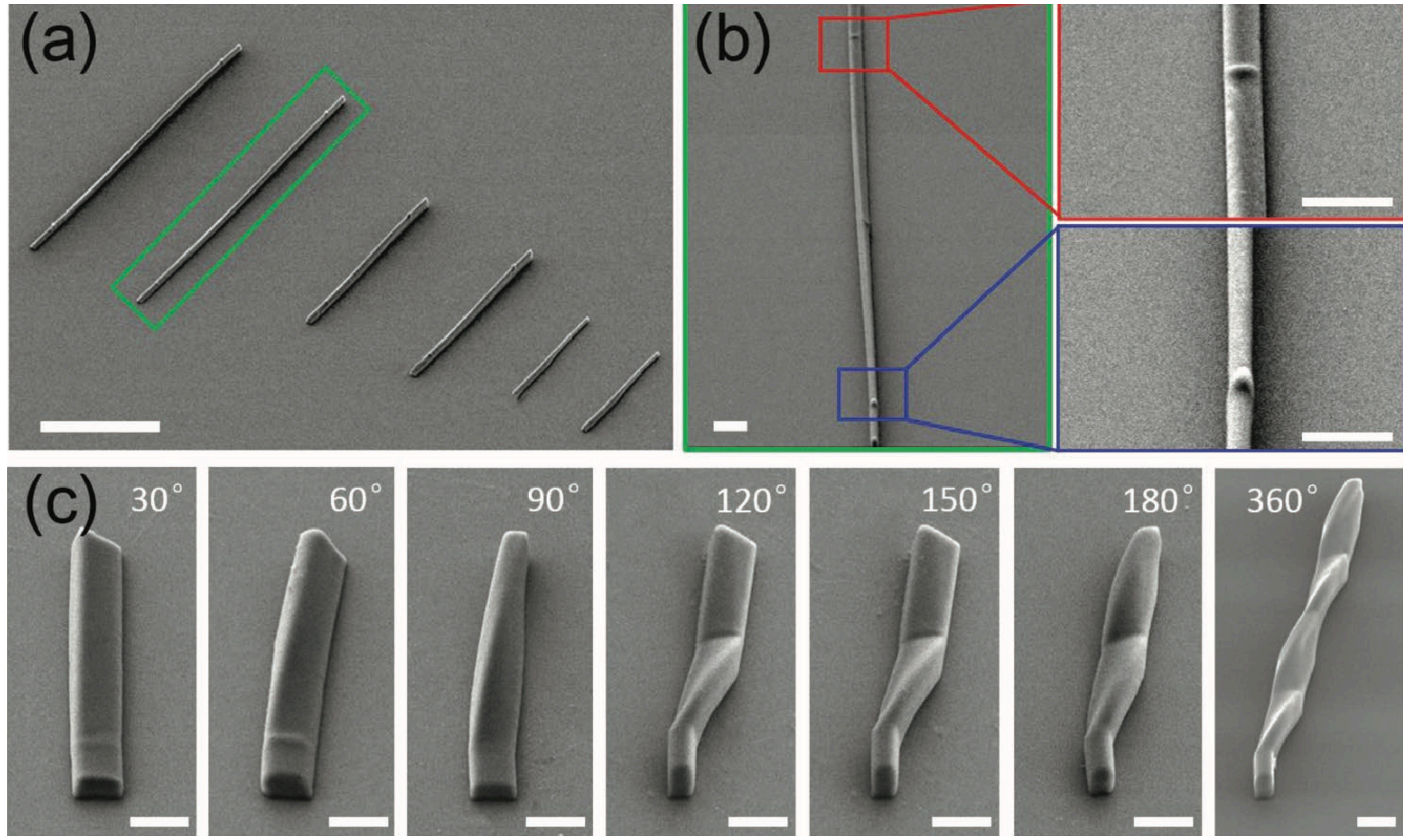


Schematic representation of the twisted waveguide based on adiabatic mode conversion. The simulated electric field distributions at the b) input, c) middle, and d) output of the twisted waveguide, with $\theta = 90°$ and $l = 200$ µm. White arrows in the center denote the polarization of the optical field. Scale bars: 1 µm.

[1] Z. Hou *et al.*, On-Chip Polarization Rotators, Advanced Optical Materials **7**, 1900129 (2019)

Polarization Rotator Based on Twisted Waveguides

- Twisted waveguides can be fabricated by **femtosecond direct laser writing**.
- Depending on the different materials and the applied laser power, the material properties may change because of polymerization, reduction, bond cleavage, phase change, and ablation.



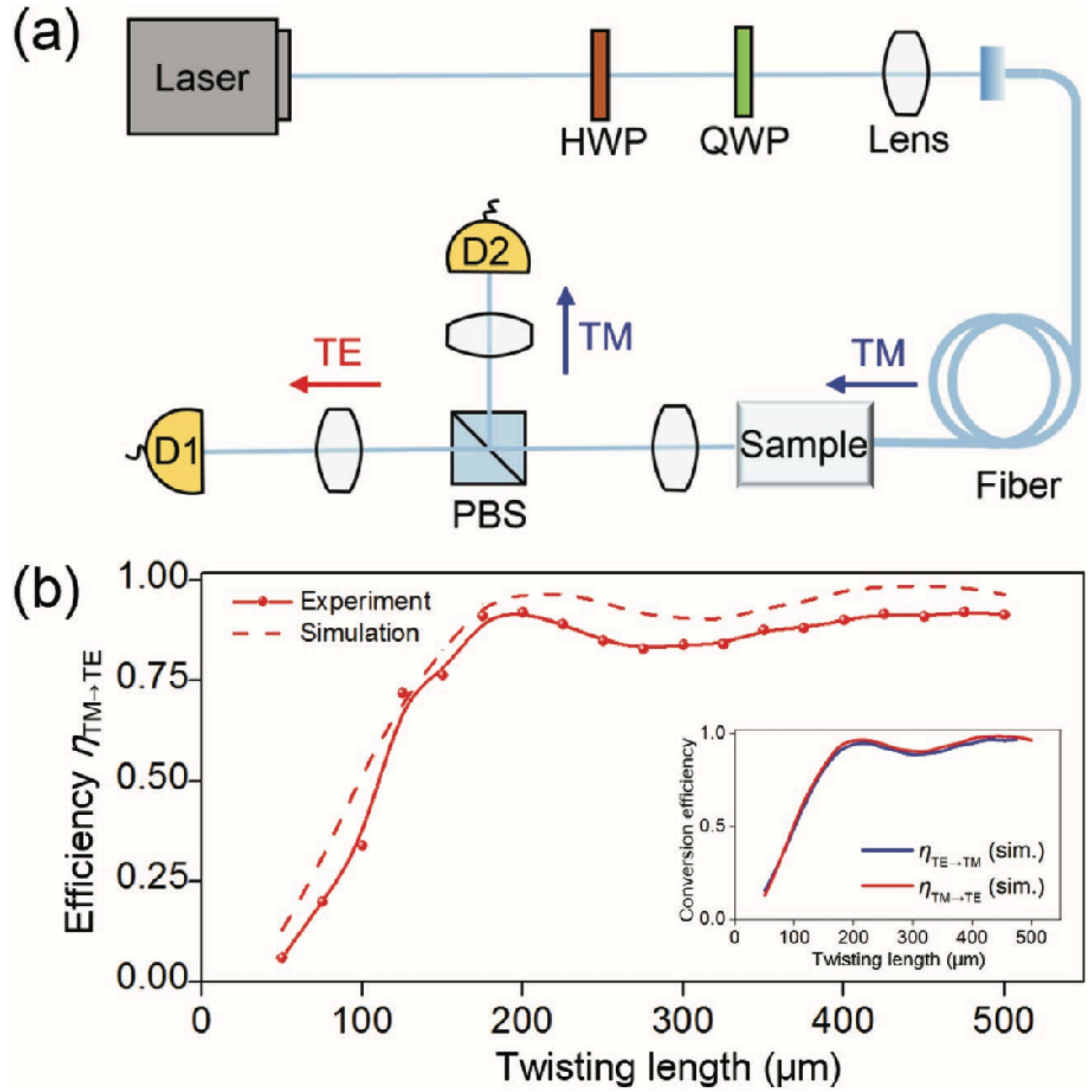
a) SEM image of the polarization rotators with twisting lengths of 150, 100, and 50 μm (from left to right). Scale bar: 100 μm.
b) Enlarged view of the sample selected as green rectangle in panel (a) with twisting length of 150 μm. Insets: zoom-in of the coupling parts. Scale bars: 10 μm.
c) SEM images of twisted segments with different twisting angles. Scale bars: 10 μm.

[1] Z. Hou *et al.*, On-Chip Polarization Rotators, Advanced Optical Materials **7**, 1900129 (2019)

Polarization Rotator Based on Twisted Waveguides

- The experimental setup to characterize the polarization rotators is displayed in the figure. The laser (λ = 1.550 nm or 646 nm) is coupled into the sample with single-mode fiber (SMF).
- Before injected into the sample, the light is tuned as TM polarization by passing through the half-wave plate (HWP) and quarter-wave plate (QWP).
- The output from the sample is collected with lens and directed into two paths with polarization beam splitter (PBS). Two power meters (D1 and D2) are used to record the TE and TM components of waveguide output, respectively.
- Denoting the counts of D1 and D2 as P_{TE} and P_{TM}, we obtain the polarization conversion efficiency expressed as

$$\eta_{\mathrm{TM}\rightarrow\mathrm{TE}} = \frac{P_{\mathrm{TE}}}{P_{\mathrm{TM}} + P_{\mathrm{TE}}}.$$



a) Experimental setup to measure the polarization rotators. HWP: half-wave plate; QWP: quarter-wave plate; PBS: polarization beam splitter; D: detector. b) The polarization conversion efficiency $\eta_{\mathrm{TM}\rightarrow\mathrm{TE}}$ for polarization rotators at λ=1.55 um with different twisting length. Inset: The polarization conversion efficiency with either TM or TE polarization input from the same input port.

[1] Z. Hou *et al.*, On-Chip Polarization Rotators, Advanced Optical Materials **7**, 1900129 (2019)

Analysis of Light Propagation in a Twisted Waveguide

Modeling Techniques for Twisted Waveguides

<table>
<tr><th>Method</th><th>Issues</th></tr>
<tr><td>Finite Difference Time Domain (FDTD)</td><td rowspan="3">➡Need for simulation in 3 dimensions
➡Very Memory Intensive (tens to hundreds GB of RAM per simulation)
➡Long Computation Times
➡Fast prototyping on a personal desktop (laptop) is problematic</td></tr>
<tr><td>Finite Difference Frequency Domain (FDFD)</td></tr>
<tr><td>Finite Element Method (FEM)</td></tr>
<tr><td>Beam Propagation Method (BPM)</td><td>➡Neglects backward-scattered field
➡Neglects z-derivatives of refractive index
➡Very CPU and RAM efficient
➡Fails to reproduce polarization rotation :(</td></tr>
<tr><td>Eigenmode Expansion Method (EME)</td><td>➡Calculates both forward and backward fields
➡Efficient for piecewise-uniform structures in propagation direction
➡Requires fine longitudinal discretization for continuously-varying structures
➡Becomes impractical for twisted waveguides</td></tr>
</table>

Utilizing Twisted Symmetry

- A twisted waveguide with constant twist rate certainly possesses some *symmetry*: its cross-section in every $z = \text{const}$ plane is constant up to rotation by a certain angle.

- If we are able to utilize this “twisted” symmetry, we can split variables in Maxwell’s equations and reduce the dimension of the problem.

- To make use of the symmetry, special coordinates can be used.

- In these coordinates we want partial derivative of $n(\mathbf{r})$ in the longitudinal direction to be zero: $\partial_Z n(X, Y, Z)\,|_{X,Y=\text{const}} = 0$.

- Thus, in analogy with the straight waveguide case we can define an **eigenmode for a twisted waveguide** with exponential, i.e. $e^{\mathrm{i}\beta Z}$ longitudinal dependence.

Twisted Coordinates

- The coordinates satisfying the requirements are:

$$X = \cos \alpha z + y \sin \alpha z,$$
$$Y = -x \sin \alpha z + y \cos \alpha z, ,$$
$$Z = z\,.$$

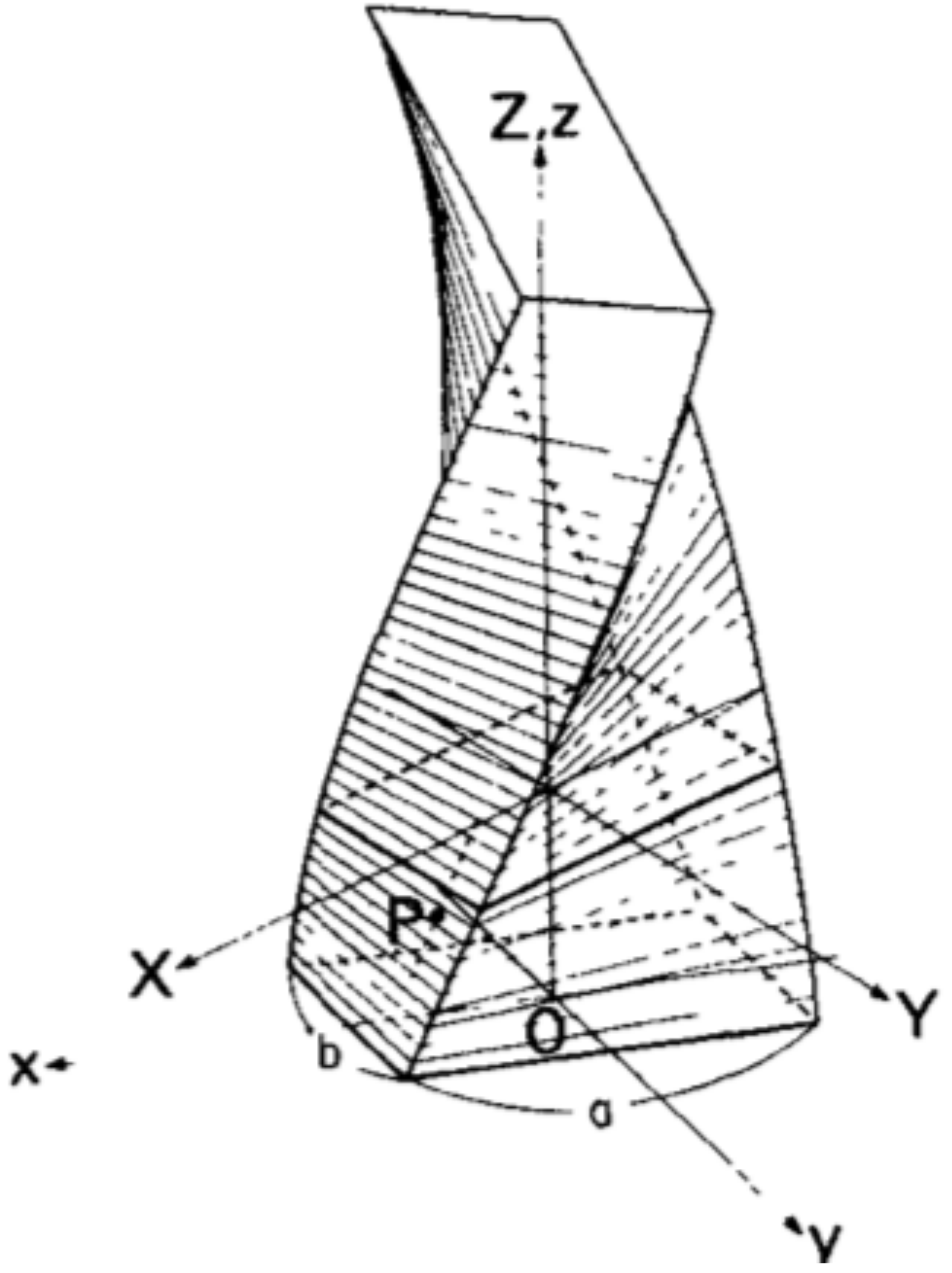
where α [rad/m] is a *twist constant*.

- We will call them **twisted coordinates**.

- Indeed, in these coordinates

$$\partial_Z n(X, Y, Z) = 0$$

just as required.



Twisted rectangular waveguide and coordinate systems [1].
Twisted (X, Y, Z) and Cartesian (x, y, z) coordinates.

[1] H. Yabe and Y. Mushiake, An Analysis of a Hybrid-Mode in a Twisted Rectangular Waveguide, IEEE Trans. Microwave Theory Techn. **32**, 65 (1984)

Twisted Coordinates

$$X = \cos \alpha z + y \sin \alpha z,$$
$$Y = -\,x \sin \alpha z + y \cos \alpha z, ,$$
$$Z = z\,.$$

- With this coordinate system two dual sets of base vecto are associated:

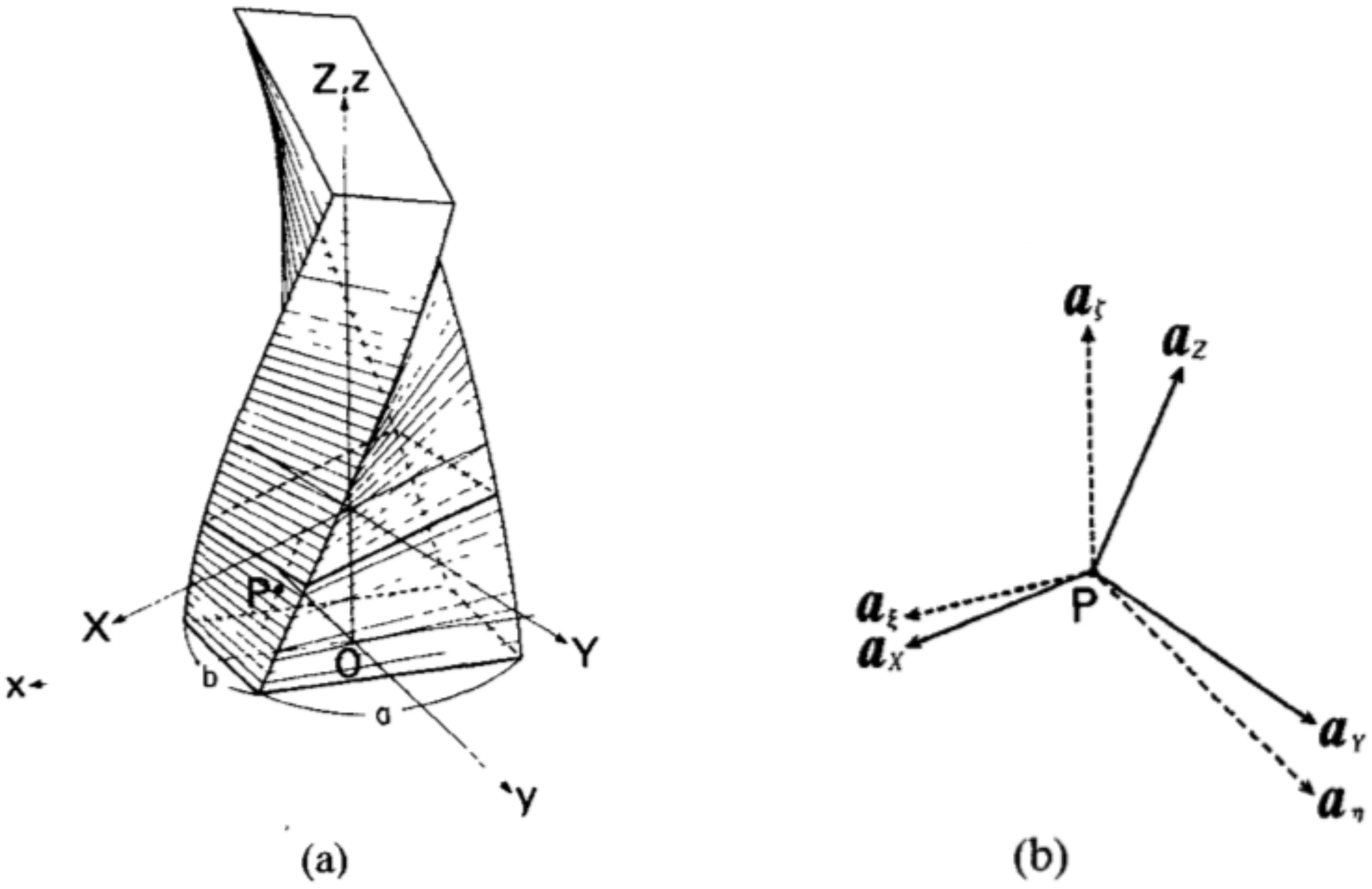
$$\mathbf{a}_X = \hat{\mathbf{x}} \cos \alpha z + \hat{\mathbf{y}} \sin \alpha z,$$
$$\mathbf{a}_Y = -\,\hat{\mathbf{x}} \sin \alpha z + +\hat{\mathbf{y}} \cos \alpha z,$$
$$\mathbf{a}_Z = -\,\alpha y \hat{\mathbf{x}} + \alpha x \hat{\mathbf{y}} + \hat{\mathbf{z}}$$

and

$$\mathbf{a}_\xi = \mathbf{a}_Y \times \mathbf{a}_Z,$$
$$\mathbf{a}_\eta = \mathbf{a}_Z \times \mathbf{a}_X,$$
$$\mathbf{a}_\zeta = \mathbf{a}_X \times \mathbf{a}_Y\,.$$

- Any vector can be expanded into both sets of bases:

$$\mathbf{v} = v_X \mathbf{a}_X + v_Y \mathbf{a}_Y + v_Z \mathbf{a}_Z = v_\xi \mathbf{a}_\xi + v_\eta \mathbf{a}_\eta + v_\zeta \mathbf{a}_\zeta.$$



Twisted rectangular waveguide and coordinate systems [1].
(a) Twisted (X, Y, Z) and Cartesian (x, y, z) coordinates.
(b) Local oblique base vectors at a point P.

[1] H. Yabe and Y. Mushiake, An Analysis of a Hybrid-Mode in a Twisted Rectangular Waveguide, IEEE Trans. Microwave Theory Techn. **32**, 65 (1984)

Maxwell Equations in Twisted Coordinates

- When working with curvilinear coordinates it is convenient to express equations in tensor notation.

- Maxwell equations for time-harmonic fields in tensor notation have the following form [1]. Here we use Gauss units:

$$
\begin{aligned}
\varepsilon^{ijk}\partial_k H_j &= -\,\mathrm{i}k_0 n^2 E^i, \\
\varepsilon^{ijk}\partial_k E_j &= \mathrm{i}k_0 H^i, \\
\partial_i n^2 E^i &= 0, \\
\partial_i H^i &= 0,
\end{aligned}
$$

where n is refractive index, ε^{ijk} is fully antisymmetric (Levi-Civita) tensor.

[1] A. McConnell, *Applications of Tensor Analysis* (Dover Publ, New York, 1957)

Eigenmodes of a Twisted Waveguide

- We can define an eigenmode for a twisted waveguide in twisted coordinates:

$$E_i(X,Y,Z) = e_i(X,Y)\mathrm{e}^{\mathrm{i}\beta Z},$$
$$H^i(X,Y,Z) = h^i(X,Y)\mathrm{e}^{\mathrm{i}\beta Z}.$$

- Substituting solution in this form into Maxwell's equations in twisted coordinates we obtain a set of equations:

$$\begin{aligned}
\mathrm{i}\beta h_Y - \partial_Y h_Z + \alpha\left(h_X + \mathrm{i}\beta X h_Z - X\partial_Y h_Y + Y\partial_X h_Y\right) + \alpha^2 X\left(Y\partial_X h_Z - X\partial_Y h_Z\right) &= -\mathrm{i}k_0 n^2 e_\xi,\\
-\mathrm{i}\beta h_X + \partial_X h_Z + \alpha\left(h_Y + \mathrm{i}\beta Y h_Z - X\partial_Y h_X + Y\partial_X h_X\right) + \alpha^2 Y\left(Y\partial_X h_Z - X\partial_Y h_Z\right) &= -\mathrm{i}k_0 n^2 e_\eta,\\
\partial_Y h_X - \partial_X h_Y - \alpha\left(2h_Z + \mathrm{i}\beta(Xh_X + Yh_Y)\right) + \alpha^2\left(Xh_Y - Yh_X + XY(\partial_Y h_Y - \partial_X h_X) + X^2\partial_Y h_X - Y^2\partial_X h_Y\right) &= -\mathrm{i}k_0 n^2 e_\zeta,\\
\mathrm{i}\beta e_\eta - \partial_Y e_\zeta &= \mathrm{i}k_0 h_X,\\
-\mathrm{i}\beta e_\xi + \partial_X e_\zeta &= \mathrm{i}k_0 h_Y,\\
\partial_Y e_\xi - \partial_X e_\eta &= \mathrm{i}k_0 h_Z
\end{aligned}$$

- These equations can be combined into an eigenmode equation with respect to e_ξ, e_η.

- There are some published works [1,2] devoted to theoretical description of eigenmodes in twisted metallic waveguides, evolution of eigenmodes in dielectric waveguides has never been described analytically.

[1] L. Lewin, Propagation in Curved and Twisted Waveguides of Rectangular Cross-Section, Proceedings of the IEE - Part B: Radio and Electronic Engineering **102**, 75 (1955)
[2] H. Yabe and Y. Mushiake, An Analysis of a Hybrid-Mode in a Twisted Rectangular Waveguide, IEEE Trans. Microwave Theory Techn. **32**, 65 (1984)

Perturbative Solutions

- Eigenmode equation for a twisted waveguide can be formally written in operator form

$$\hat{H}(\beta)\psi = 0,$$

where $\psi \equiv (e_\xi, e_\eta)$ and $\hat{H}$ is the differential operator obtained from the Maxwell's equations in twisted coordinates.

- $\hat{H}$ has the form

$$\hat{H} = \hat{H}^{(0)} + \alpha\hat{H}^{(1)} + \alpha^2\hat{H}^{(2)},$$

where $\hat{H}^{(0)}$ is the operator coinciding with the eigenmode equation operator in the absence of twist.

- Assuming that the twist constant α is small (in sense $\alpha/k_0 \ll 1$) we can expand modal fields and propagation constants in series of α

$$\psi = \psi^{(0)} + \alpha\psi^{(1)} + \alpha^2\psi^{(2)} + \ldots$$

- As was firstly pointed by Lewin [1], propagation constant can only contain even powers of α in the expansion, because the twist direction does not affect it, so

$$\beta = \beta^{(0)} + \alpha^2\beta^{(2)} + \ldots$$

[1] L. Lewin, Propagation in Curved and Twisted Waveguides of Rectangular Cross-Section, Proceedings of the IEE - Part B: Radio and Electronic Engineering **102**, 75 (1955)

Perturbative Solutions

- For the unperturbed system corresponding to a straight waveguide

$$\hat{H}^{(0)}\psi_m^{(0)} = \beta_m^{(0)2}\psi_m^{(0)},$$

where $\psi \equiv (e^i, h^i)^T$ and $\hat{H}$ is the differential operator obtained from the Maxwell's equations in twisted coordinates.

- The exact solution is expanded into an orthogonal set of unperturbed modes i.e. modes of straight waveguide:

$$\psi_m = \sum_m C_{m\tilde{m}}\psi^{(0)}.$$

- By using orthogonality relations of unperturbed modes we can find the exact equations for the expansion coefficients in terms of matrix elements of perturbation operators

$$\left(\beta_m - \beta_k^{(0)}\right) C_{nk} = \sum_m (\alpha H_{km}^{(1)} + \alpha^2 H_{km}^{(2)}) C_{nm},$$

$$H_{km}^{(1,2)} = \langle \psi_k^{(0)} | \hat{H}^{(1,2)} | \psi_m^{(0)} \rangle, \text{ where } \langle \psi_1 | \psi_2 \rangle = \frac{1}{2}\int (\mathbf{e}_1 \times \mathbf{h}_2^*)_z dxdy.$$

- The perturbation theory corresponds to the expansion of values in question into series over α:

$$C_{nk} = C_{nk}^{(0)} + \alpha C_{nk}^{(1)} + \alpha^2 C_{nk}^{(2)} + \ldots$$

[1] I. Feranchuk, *et al., Non-Perturbative Description of Quantum Systems* (Springer Berlin Heidelberg, New York, 2015)

"Twisted" Eigenmode Expansion Method

- Provided that we have found the eigenmodes of a twisted waveguide we can formulate a **modified EME method** suitable for twisted waveguides.
- Within this method, the total field in the twisted waveguide is expressed as

$$\mathbf{E}(X, Y, Z) = \sum_m A_m \mathbf{e}_m(X, Y) e^{\mathrm{i}\beta_m Z} + \sum_m B_m \mathbf{e}_{-m}(X, Y) e^{-\mathrm{i}\beta_m Z},$$

where $\mathbf{e}_m \equiv e_m^k$ are fields of twisted waveguide modes, A_m, B_m are complex amplitudes.

“Twisted” Eigenmode Expansion Method

- Let us assume that a twisted waveguide is facet-coupled to a straight waveguide at $Z = 0$. And a single mode with the field $\mathbf{E}^{(0)} = \mathbf{e}^{(0)} \exp(i\beta^{(0)} Z)$ is launched into the straight waveguide.

- The continuity of transverse fields requires

$$\mathbf{E}_t^{(0)}(X, Y, 0-) = \mathbf{E}_t(X, Y, 0+),\ \mathbf{H}_t^{(0)}(X, Y, 0-) = \mathbf{H}_t(X, Y, 0+).$$

- Substituting modal expansion for the field in the twisted waveguide we obtain

$$\mathbf{e}_t^{(0)}(X, Y) = \sum_m A_m \mathbf{e}_{m,t}(X, Y) + \sum_m B_m \mathbf{e}_{-m,t}(X, Y),$$

$$\mathbf{h}_t^{(0)}(X, Y) = \sum_m A_m \mathbf{h}_{m,t}(X, Y) + \sum_m B_m \mathbf{h}_{-m,t}(X, Y)\,.$$

- We can expand the modal fields of the twisted waveguide in terms of modes of the straight waveguide:

$$\mathbf{e}_{m,t} = \sum_{\tilde{m}} C_{m\tilde{m}} \mathbf{e}^{(0)}_{\tilde{m},t},\ \mathbf{h}_{m,t} = \sum_{\tilde{m}} C_{m\tilde{m}} \mathbf{h}^{(0)}_{\tilde{m},t}$$

with expansion coefficients obtained by perturbation approach.

- By substituting expansions of the “twisted” modes in terms of “untwisted” ones and applying the orthogonality of the “untwisted” modes $\mathbf{e}_m^{(0)}$ we find the amplitudes A_m, B_m.

Issues

- Definition of eigenmode are possible in terms of co- and contravariant vectors $e_i(X, Y), e^i(X, Y)$ and so with h^i.
- So there are at least 4 possible variants and this is up to us to chose from them. Each variant has its advantages and disadvantages.
- This number is multiplied by several possible ways to formulate the eigenmode equation from Maxwell's equations.
- The perturbational corrections are expressed in terms of cumbersome integro-differential forms of unperturbed fields.
- Radiation modes must also contribute to the expansions of twisted modes. But it is a challenge to take radiation modes into account when finding eigenmodes numerically.

Summary

✓ Twisted waveguide is a promising platform for realization of polarization rotators.

✓ Although they can be modeled with full-wave 3D numerical approaches, there is demand of a more efficient and insightful analytical modeling method.

✓ We propose a method based on expansion of the field in terms of eigenmodes of a twisted waveguide.

✓ The modes of twisted waveguides can be found by perturbation theory as expansion in terms of modes of a straight waveguide.

✓ The perturbation approach allows to use existing mode-solving software to obtain modes of twisted waveguide.